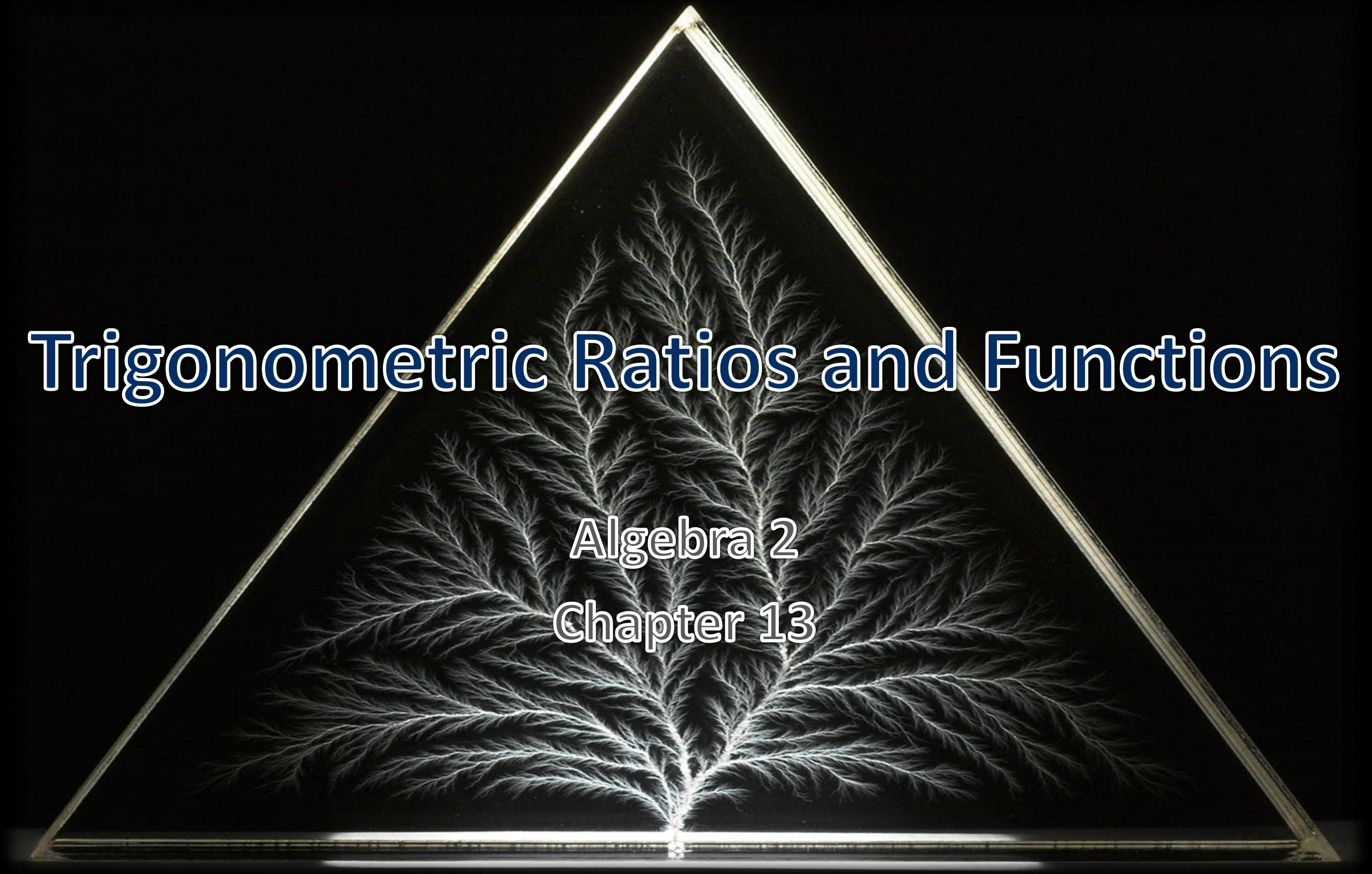

Trigonometric Ratios and Functions

Algebra 2

Chapter 13

- This Slideshow was developed to accompany the textbook
    - *Larson Algebra 2*
    - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
    - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

Slides created by
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13.1 Use Trigonometry with Right Triangles

- If you have a right triangle, there are six ratios of sides that are always constant
  - $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
  - $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
  - $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
  - $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$
  - $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$
  - $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

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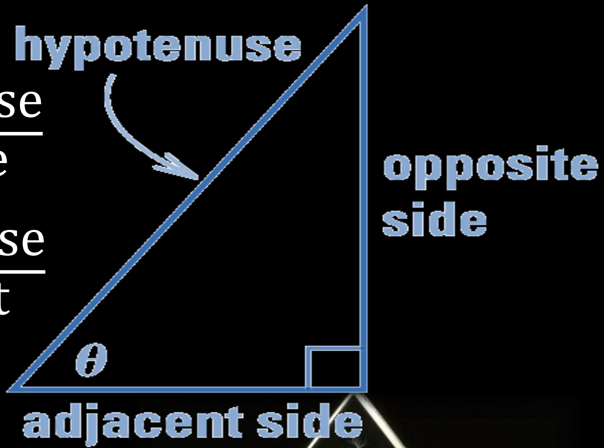
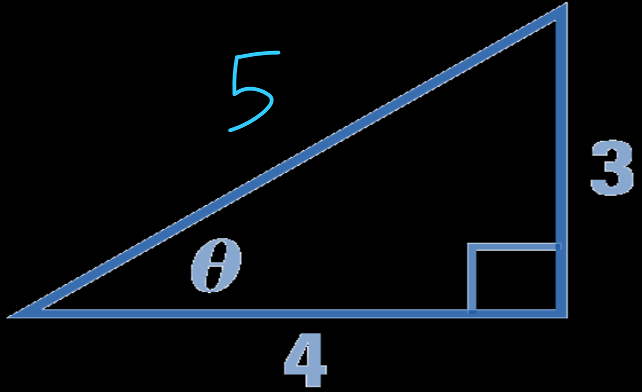


13.1 Use Trigonometry with Right Triangles

- Evaluate the six trigonometric functions of the angle θ.

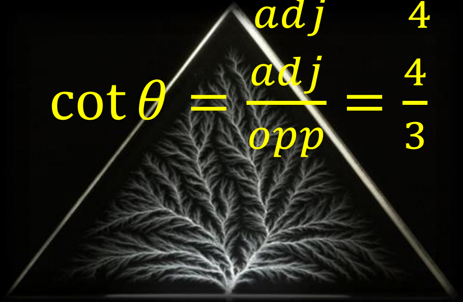


- Use Pythagorean Theorem to find hypotenuse
- $3^2 + 4^2 = hyp^2$
- $hyp = 5$
- $\sin\theta = \frac{opp}{hyp} = \frac{3}{5}$
- $\cos\theta = \frac{adj}{hyp} = \frac{4}{5}$
- $\tan\theta = \frac{opp}{adj} = \frac{3}{4}$

$\csc\theta = \frac{hyp}{opp} = \frac{5}{3}$

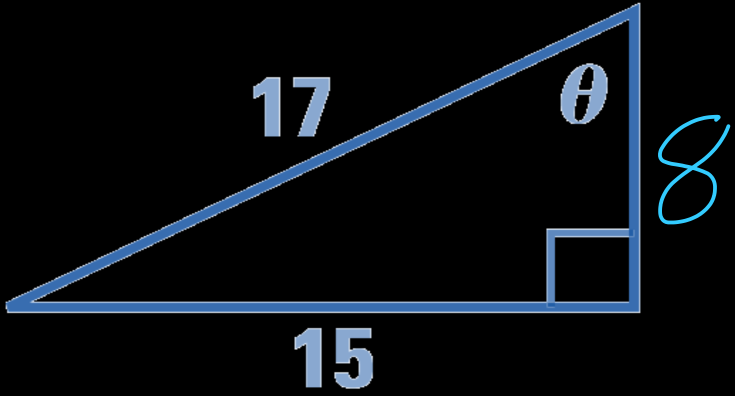
$\sec\theta = \frac{hyp}{adj} = \frac{5}{4}$

$\cot\theta = \frac{adj}{opp} = \frac{4}{3}$

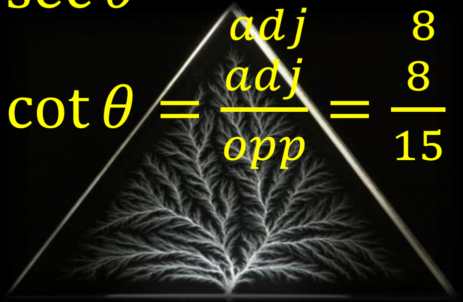


13.1 Use Trigonometry with Right Triangles

- Evaluate the six trigonometric functions of the angle θ.

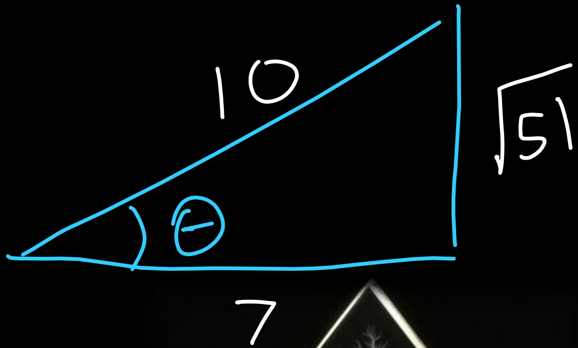


- Use Pythagorean Theorem to find adjacent
- $15^2 + adj^2 = 17^2$
- $adj = 8$
- $\sin\theta = \frac{opp}{hyp} = \frac{15}{17}$ $\quad \csc\theta = \frac{hyp}{opp} = \frac{17}{15}$
- $\cos\theta = \frac{adj}{hyp} = \frac{8}{17}$ $\quad \sec\theta = \frac{hyp}{adj} = \frac{17}{8}$
- $\tan\theta = \frac{opp}{adj} = \frac{15}{8}$ $\quad \cot\theta = \frac{adj}{opp} = \frac{8}{15}$



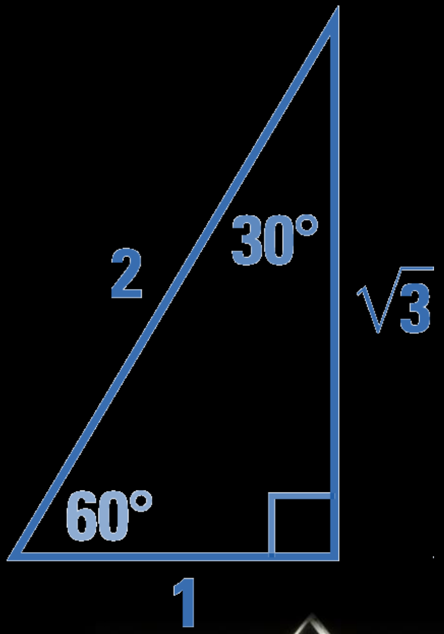
13.1 Use Trigonometry with Right Triangles

- In a right triangle, θ is an acute angle and $\cos\theta = \frac{7}{10}$. What is $\sin\theta$?
- Draw the triangle and pick an acute angle for θ.
- Label the adj 7 and hyp 10
- Use Pythagorean Theorem to find opp.
  - $7^2 + opp^2 = 10^2$
  - $opp = \sqrt{51}$
- $\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{51}}{10}$

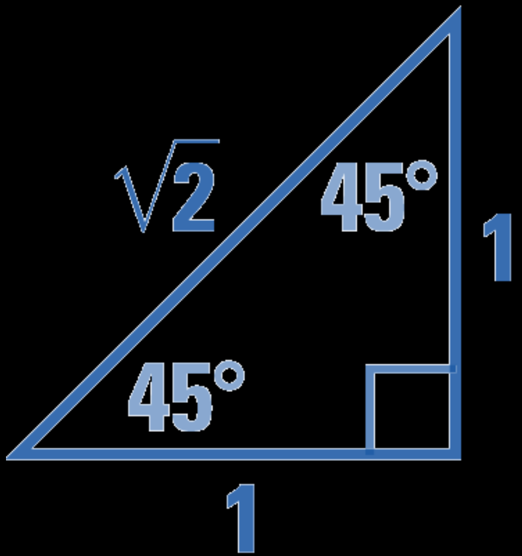


13.1 Use Trigonometry with Right Triangles

- Special Right Triangles
- 30° - 60° - 90°

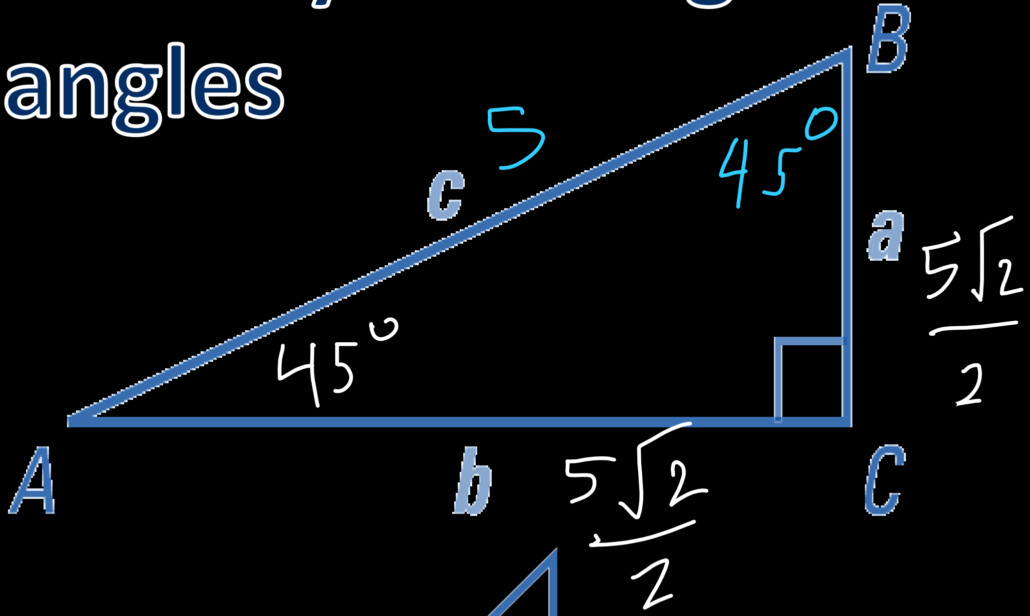


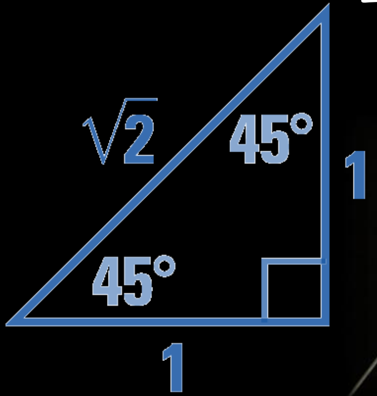
- 45° - 45° - 90°



13.1 Use Trigonometry with Right Triangles

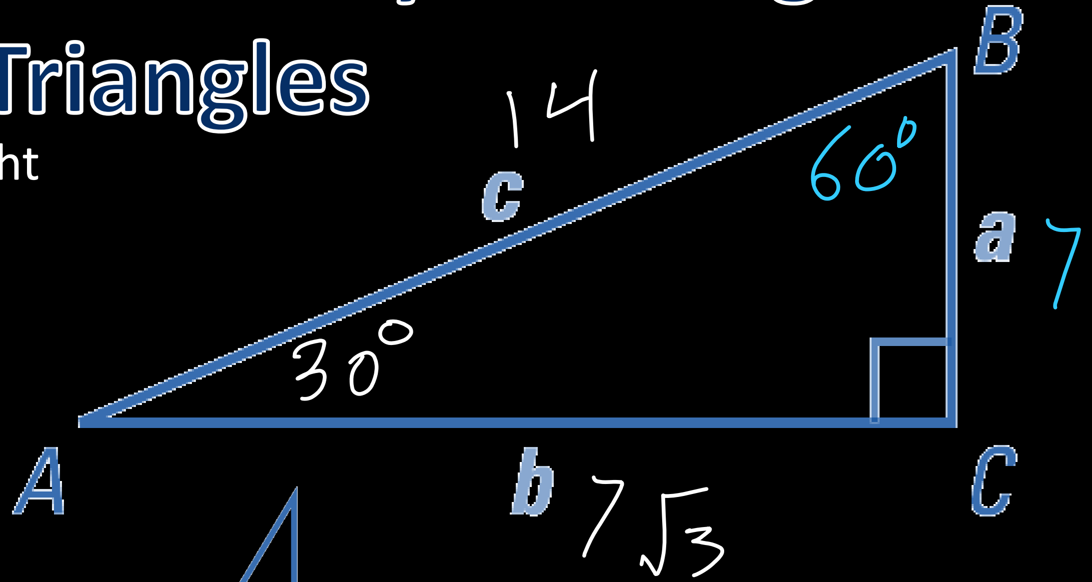
- Use the diagram to solve the right triangle if…
- B = 45°, c = 5
- $A = 90° - 45° = 45°$
- From special rt triangle,
  - $\cos 45° = \frac{a}{5} = \frac{1}{\sqrt{2}}$
    - $a = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$
  - $\sin 45° = \frac{b}{5} = \frac{1}{\sqrt{2}}$
    - $b = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

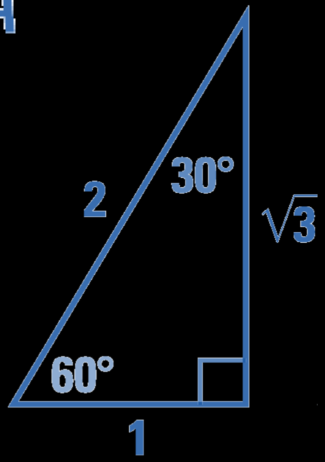




13.1 Use Trigonometry with Right Triangles

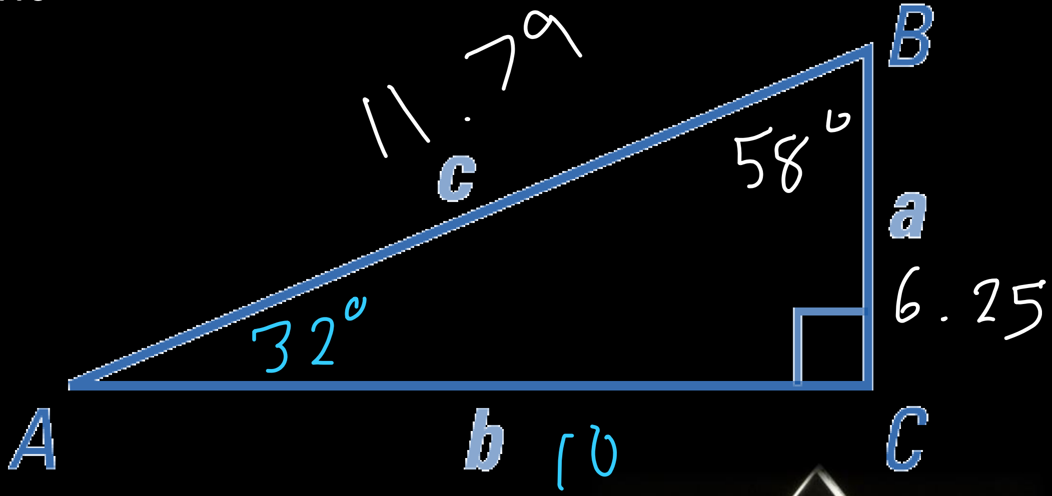
- Use the diagram to solve the right triangle if…
- B = 60°, a = 7
- $A = 90° - 60° = 30°$
- From special rt triangle;
- $\tan 60° = \frac{b}{7} = \frac{\sqrt{3}}{1}$
  - $b = 7\sqrt{3}$
- $\cos 60° = \frac{7}{c} = \frac{1}{2}$
  - $c = 14$





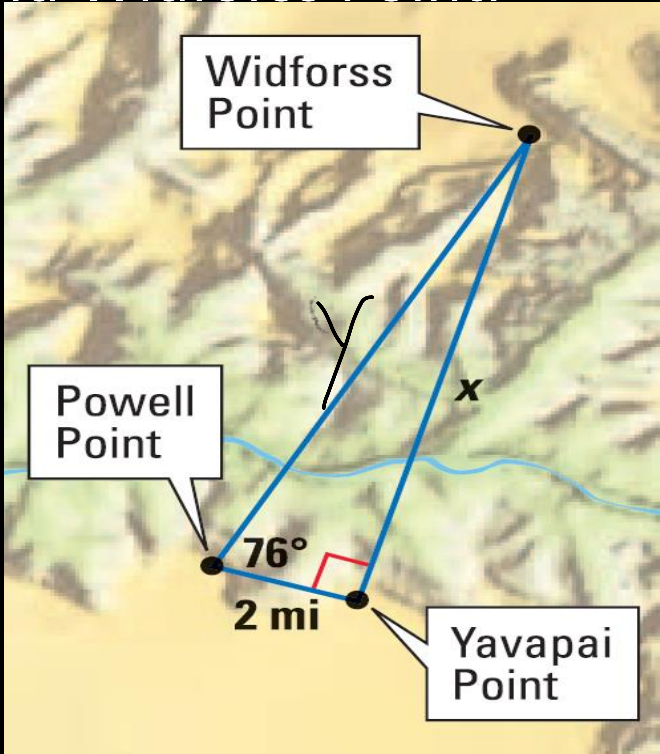
13.1 Use Trigonometry with Right Triangles

- Use the diagram to solve the right triangle if…
- A = 32°, b = 10
- $B = 90° - 32° = 58°$
- $\tan 32° = \frac{a}{10}$
  - $a = 10 \tan 32° \approx 6.25$
- $\cos 32° = \frac{10}{c}$
  - $c \cdot \cos 32° = 10$
  - $c = \frac{10}{\cos 32°} \approx 11.79$



13.1 Use Trigonometry with Right Triangles

- Find the distance between Powell Point and Widforss Point.
- $\cos 76° = \frac{2 \text{ mi}}{y}$
- $y \cdot \cos 76° = 2 \text{ mi}$
- $y = \frac{2 \text{ mi}}{\cos 76°} \approx 8.27 \text{ mi}$

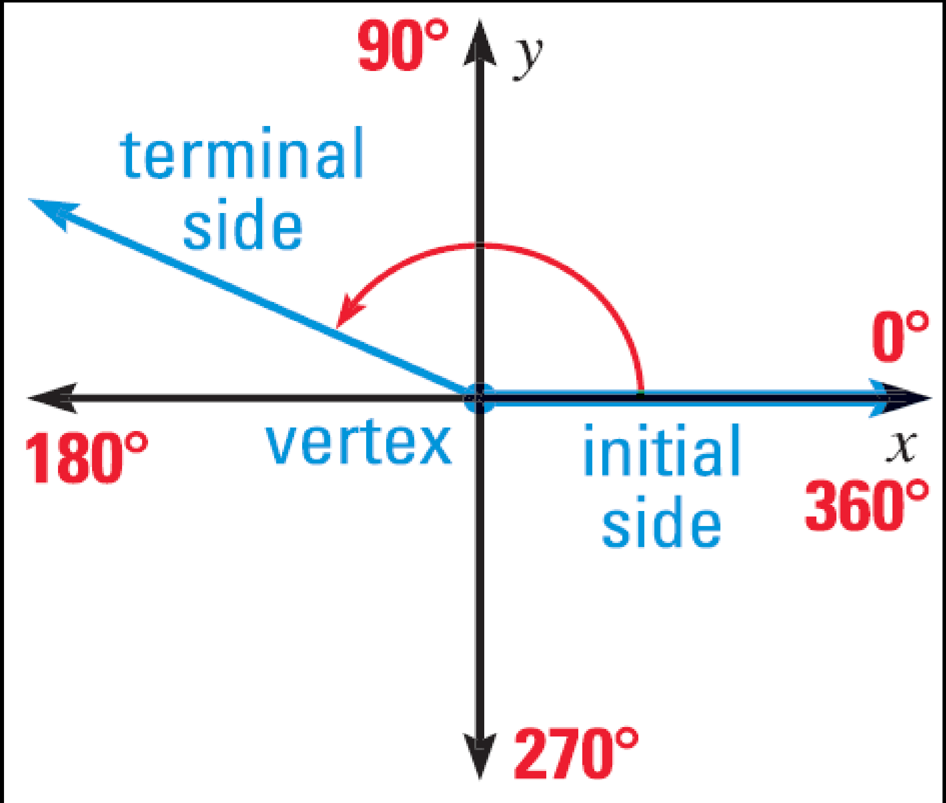


Quiz

- 13.1 Homework Quiz

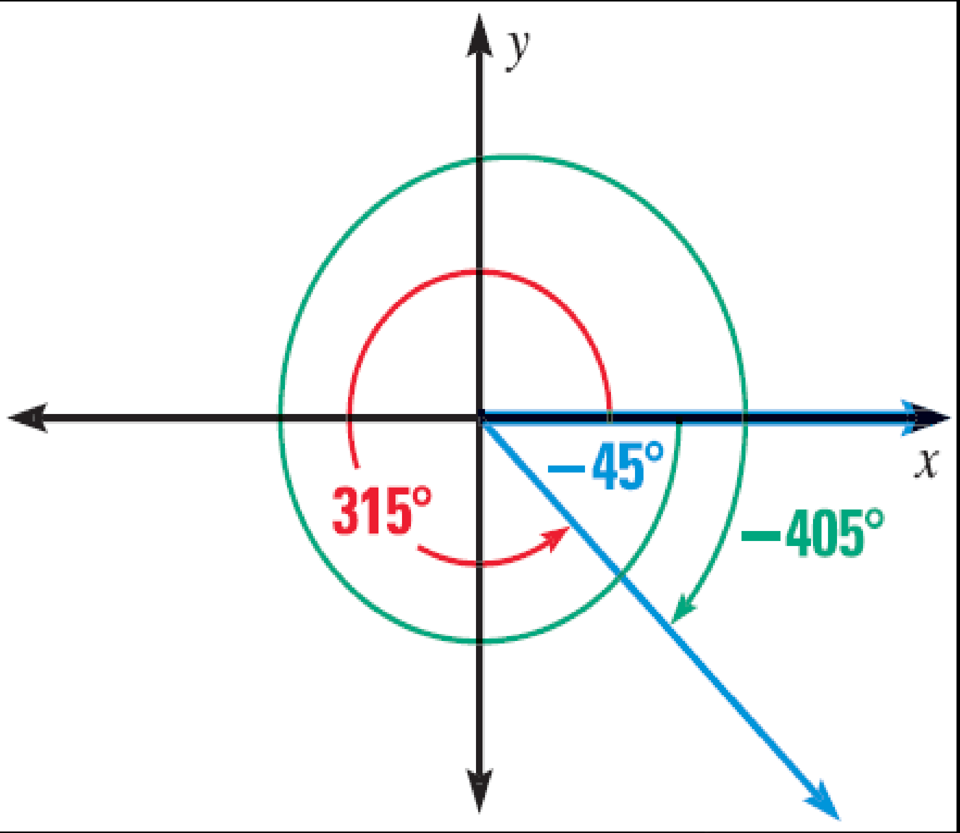
13.2 Define General Angles and Use Radian Measure

- Angles in Standard Position
  - Vertex on origin
  - Initial Side on positive x-axis
  - Measured counterclockwise



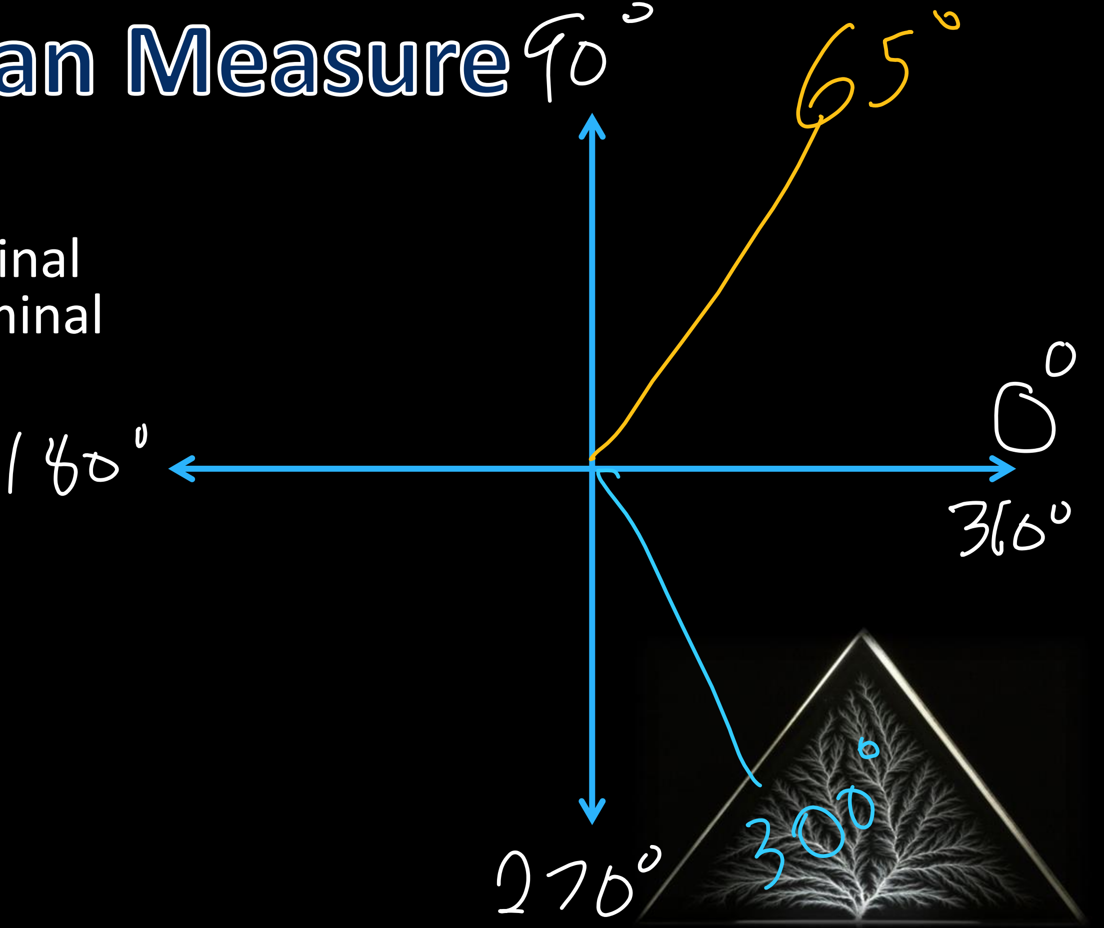
13.2 Define General Angles and Use Radian Measure

- Coterminal Angles
  - Different angles (measures) that have the same terminal side
  - Found by adding or subtracting multiples of 360°



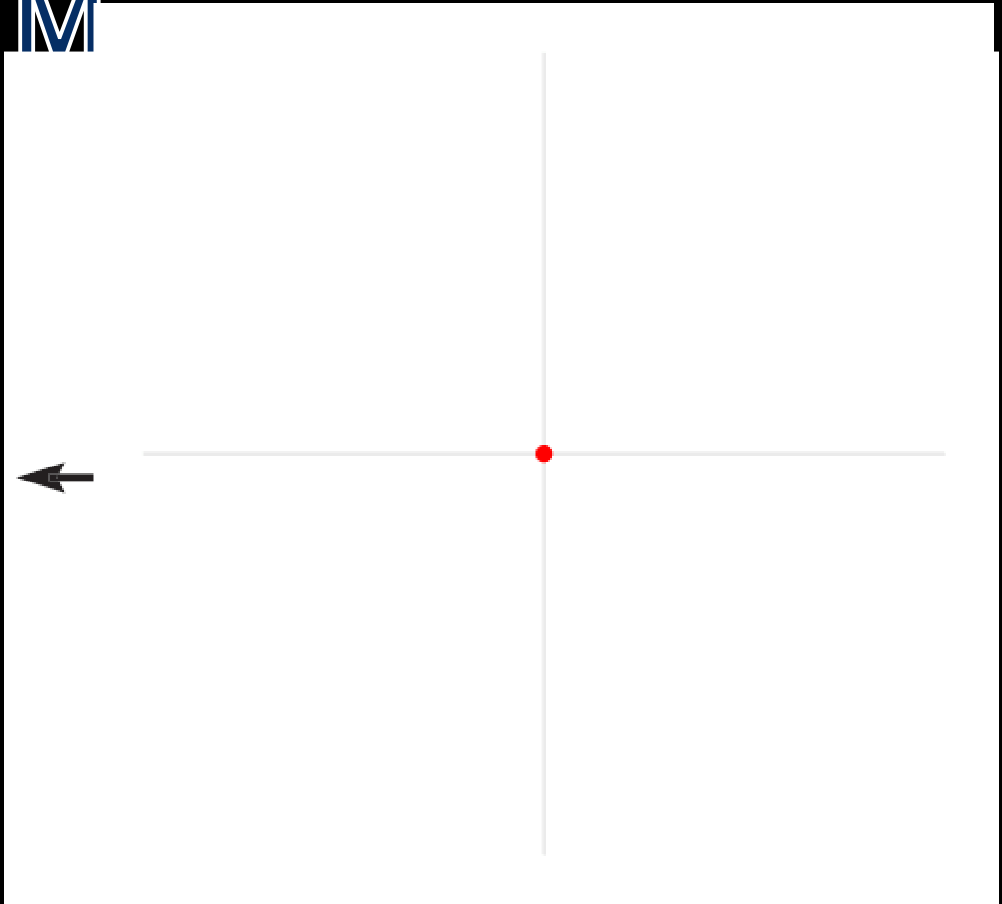
13.2 Define General Angles and Use Radian Measure

- Draw an angle with the given measure in standard position. Then find one positive coterminal angle and one negative coterminal angle.
- 65°
- $65° + 360° = 425°$
- $65° - 360° = -295°$
- 300°
- $300° + 360° = 660°$
- $300° - 360° = -60°$



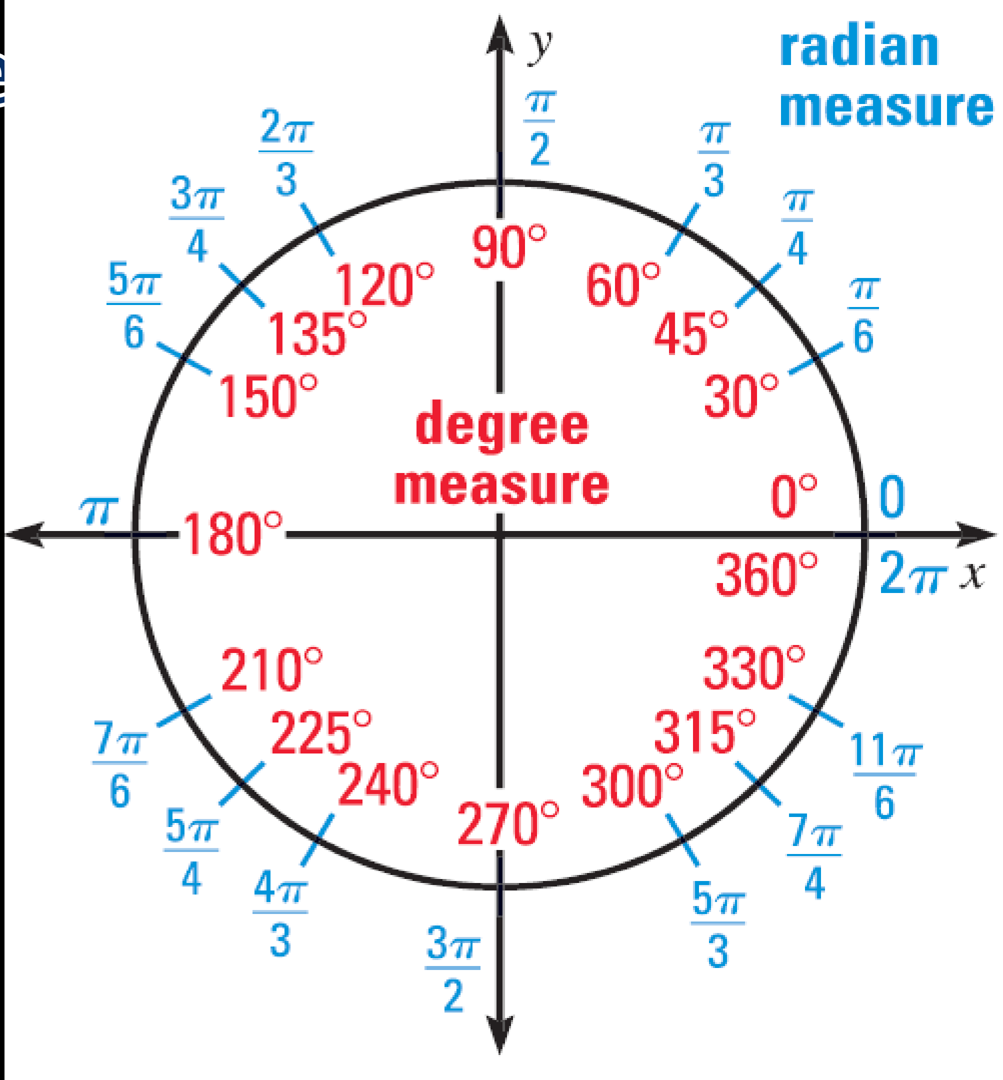
13.2 Define General Angles and Use Radian M

- Radian measure
  - Another unit to measure angles
  - 1 radian is the angle when the arc length = the radius
  - There are 2π radians in a circle



13.2 Define Gene
Radian

- To convert between degrees and radians use fact that
  - 180° = π
- Special angles

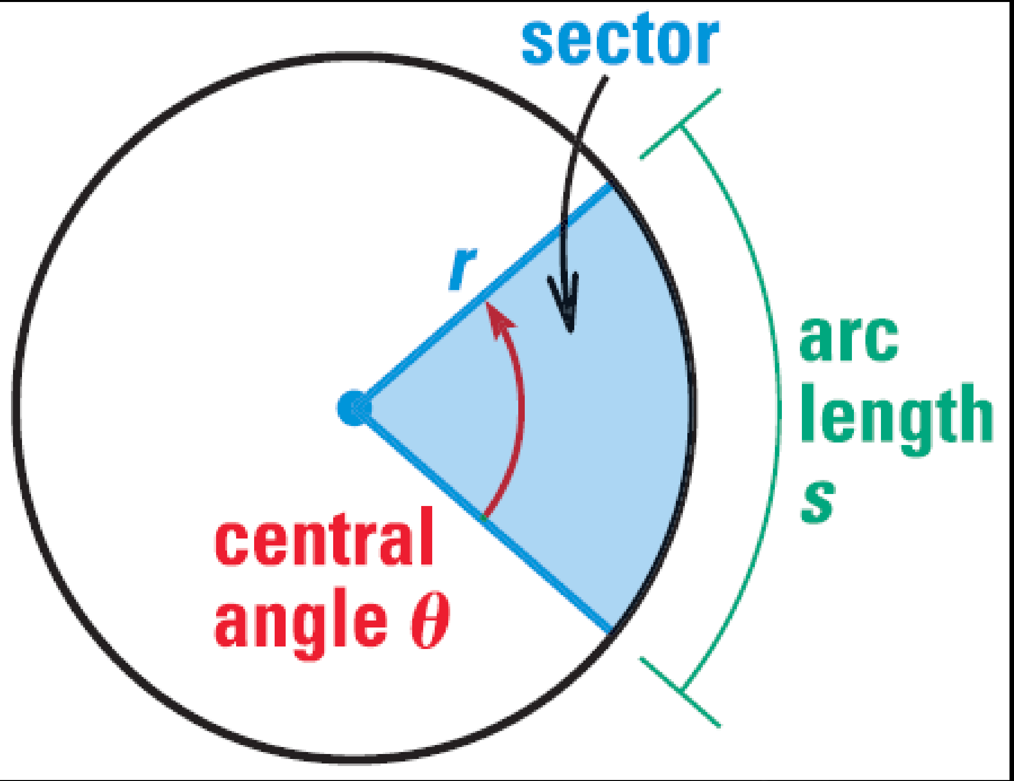


13.2 Define General Angles and Use Radian Measure

- Convert the degree measure to radians, or the radian measure to degrees.
- 135°
  - $135°\left(\frac{\pi}{180°}\right) = \frac{3\pi}{4}$
- -50°
  - $-50°\left(\frac{\pi}{180°}\right) = -\frac{5\pi}{18}$
- $\frac{5\pi}{4}$
  - $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$
- $\frac{\pi}{10}$
  - $\frac{\pi}{10}\left(\frac{180°}{\pi}\right) = 18°$

13.2 Define General Angles and Use Radian Measure

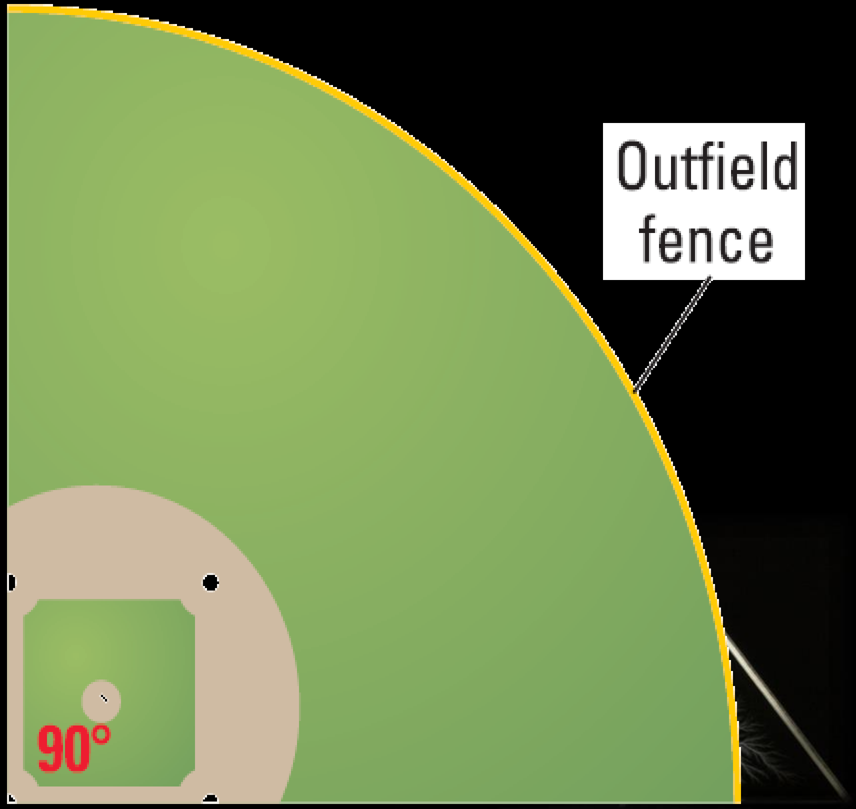
- Sector
  - Slice of a circle
- Arc Length
  - $s = r\theta$
  - θ must be in radians!
- Area of Sector
  - $A = \frac{1}{2}r^2\theta$
  - θ must be in radians!



13.2 Define General Angles and Use Radian Measure

- Find the length of the outfield fence if it is 220 ft from home plate.
- $\theta = \frac{\pi}{2}$
- $s = r\theta$
- $s = 220\left(\frac{\pi}{2}\right)$
- $s = 110\pi \approx 346\ ft$

- Find the area of the baseball field.
- $A = \frac{1}{2}r^2\theta$
- $A = \frac{1}{2}(220)^2\left(\frac{\pi}{2}\right)$
- $A = 12100\pi \approx 38013 ft^2$

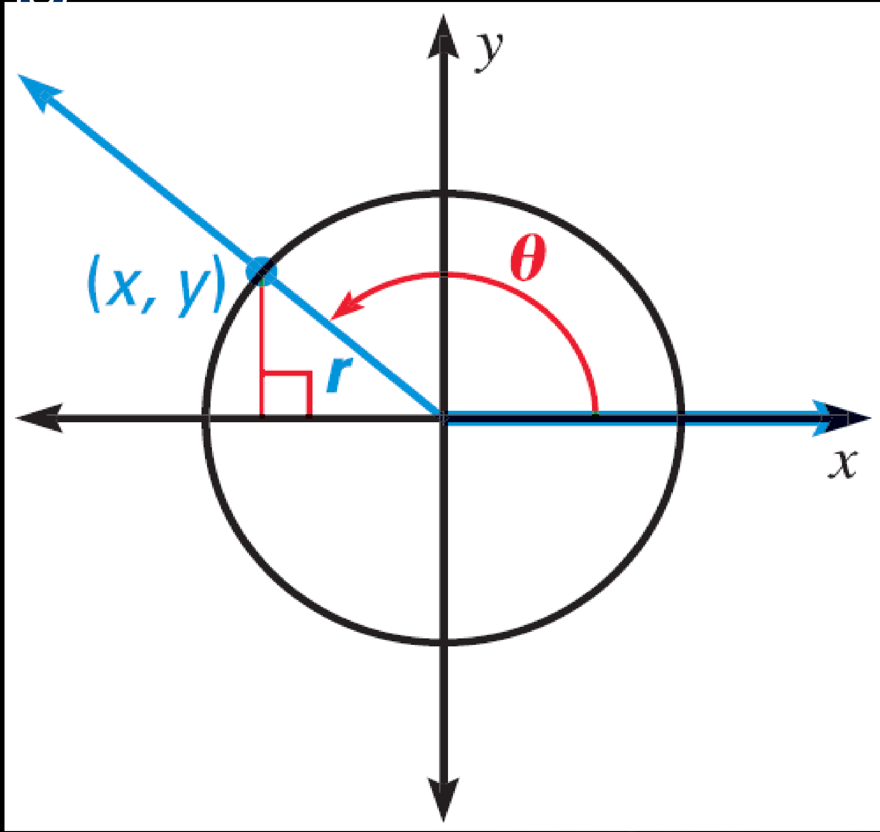


Quiz

- 13.2 Homework Quiz

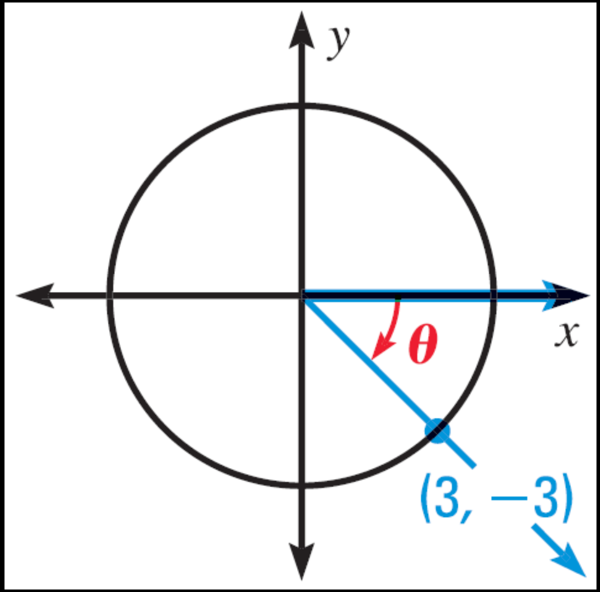
13.3 Evaluate Trigonometric Functions of Any Angle

- Think of a point on the terminal side of an angle
- You can draw a right triangle with the x-axis
- $\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$
- $\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$
- $\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$
- Unit Circle
  - r = 1



13.3 Evaluate Trigonometric Functions of Any Angle

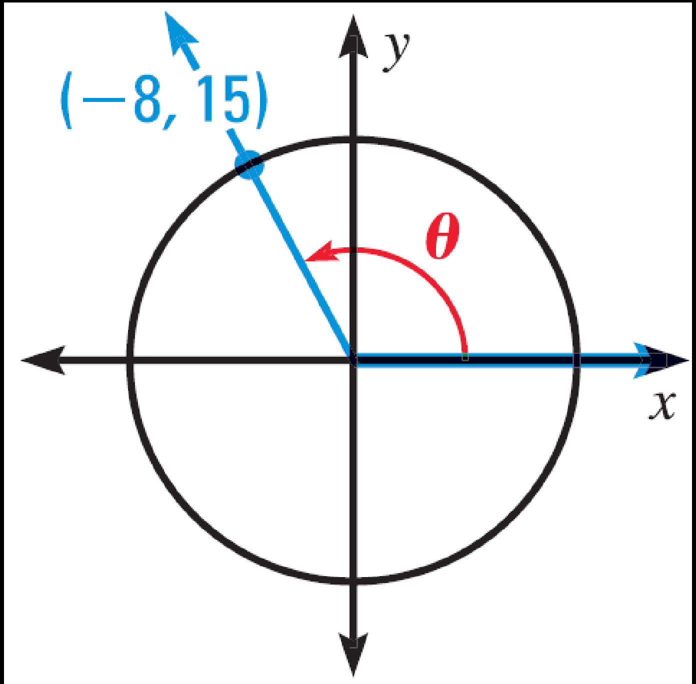
- Evaluate the six trigonometric functions of θ.



- $r^2 = x^2 + y^2$
- $r^2 = 3^2 + (-3)^2$
- $r = \sqrt{18} = 3\sqrt{2}$
- $\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{2}} = -\frac{\sqrt{2}}{2}$
- $\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $\tan\theta = \frac{y}{x} = \frac{-3}{3} = -1$
- $\csc\theta = \frac{r}{y} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$
- $\sec\theta = \frac{r}{x} = \frac{3\sqrt{2}}{3} = \sqrt{2}$
- $\cot\theta = \frac{x}{y} = \frac{3}{-3} - 1$

13.3 Evaluate Trigonometric Functions of Any Angle

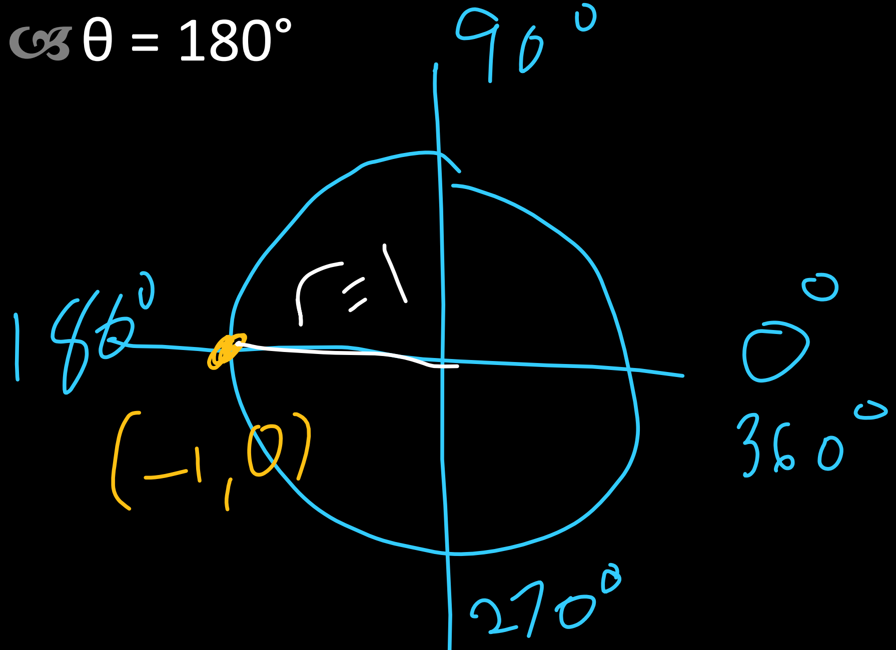
- Evaluate the six trigonometric functions of θ.



- $r^2 = x^2 + y^2$
- $r^2 = (-8)^2 + 15^2 = 289$
- $r = 17$
- $\sin\theta = \frac{y}{r} = \frac{15}{17}$
- $\cos\theta = \frac{x}{r} = -\frac{8}{17}$
- $\tan\theta = \frac{y}{x} = -\frac{15}{8}$
- $\csc\theta = \frac{r}{y} = \frac{17}{15}$
- $\sec\theta = \frac{r}{x} = -\frac{17}{8}$
- $\cot\theta = \frac{x}{y} = -\frac{8}{15}$

13.3 Evaluate Trigonometric Functions of Any Angle

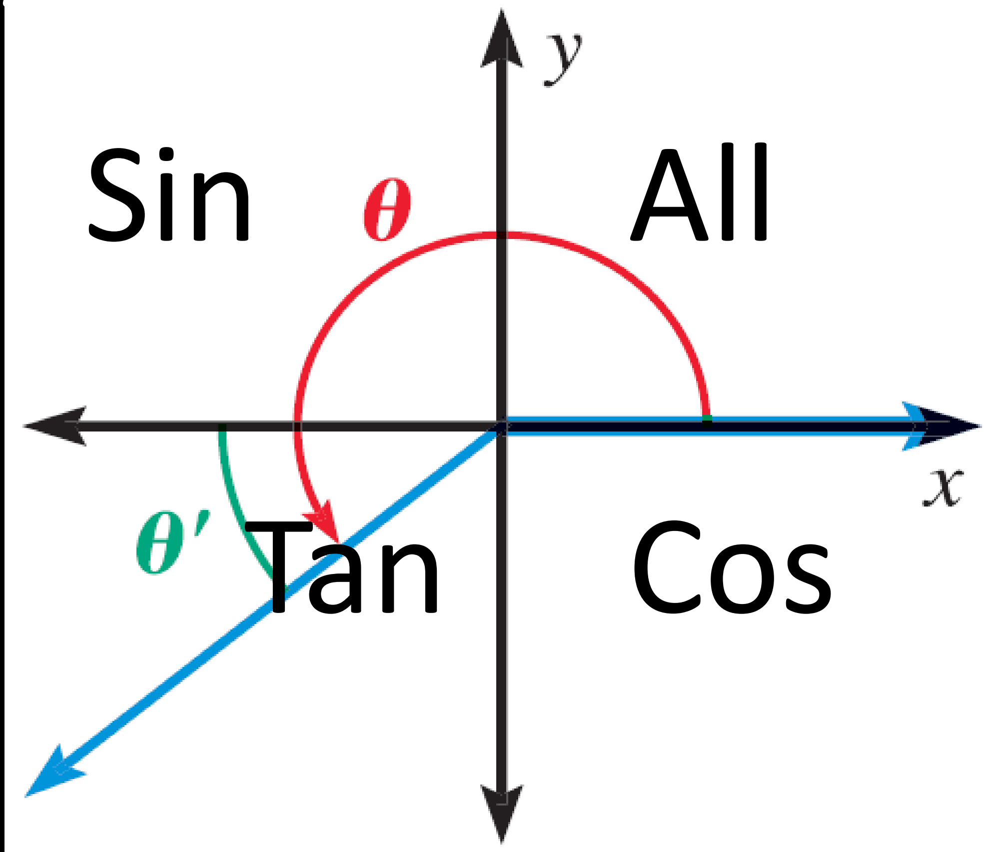
- Evaluate the six trigonometric functions of θ.
- θ = 180°



- $\sin\theta = \frac{y}{r} = \frac{0}{1} = 0$
- $\cos\theta = \frac{x}{r} = \frac{-1}{1} = -1$
- $\tan\theta = \frac{y}{x} = \frac{0}{-1} = 0$
- $\csc\theta = \frac{r}{y} = \frac{1}{0} = und$
- $\sec\theta = \frac{r}{x} = \frac{1}{-1} = -1$
- $\cot\theta = \frac{x}{y} = \frac{-1}{0} = und$

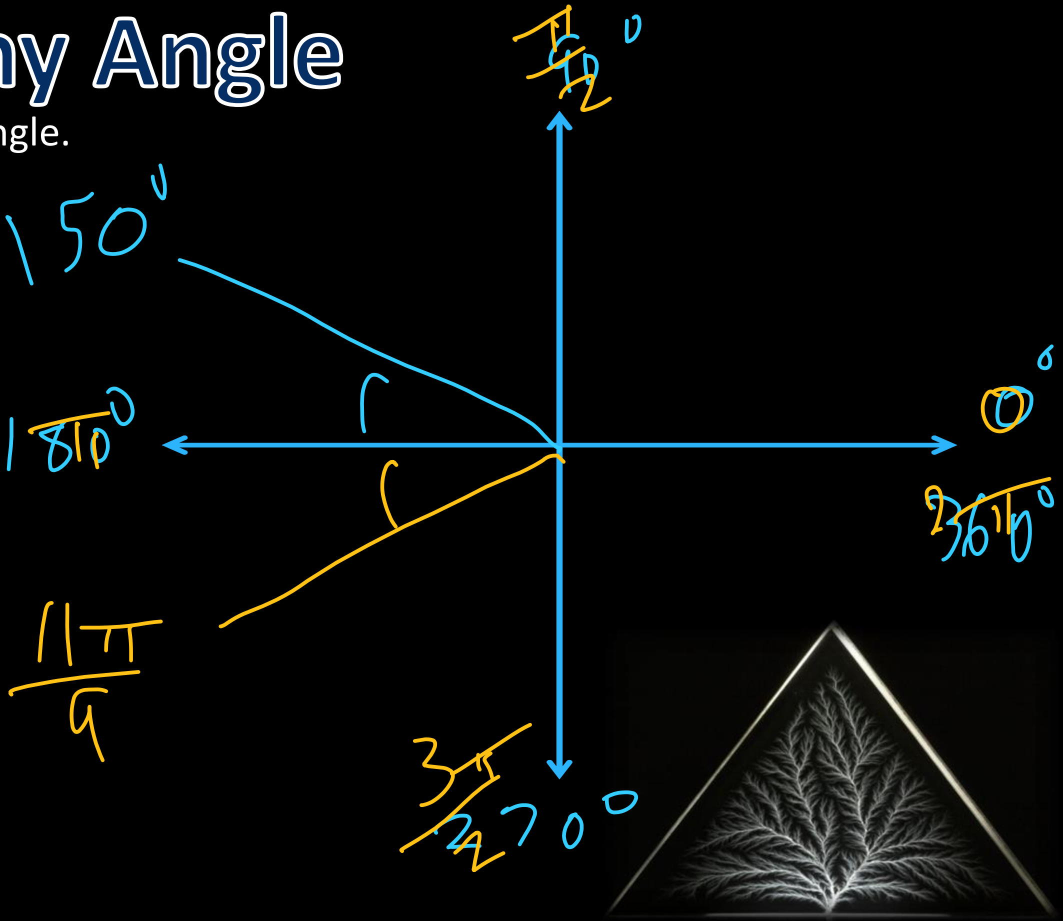
13.3 Evaluate Trigonometric Functions of Any Angle

- Reference Angle
  - Angle between terminal side and x-axis
  - Has the same values for trig functions as 1st quadrant angles
  - You just have to add the negative signs



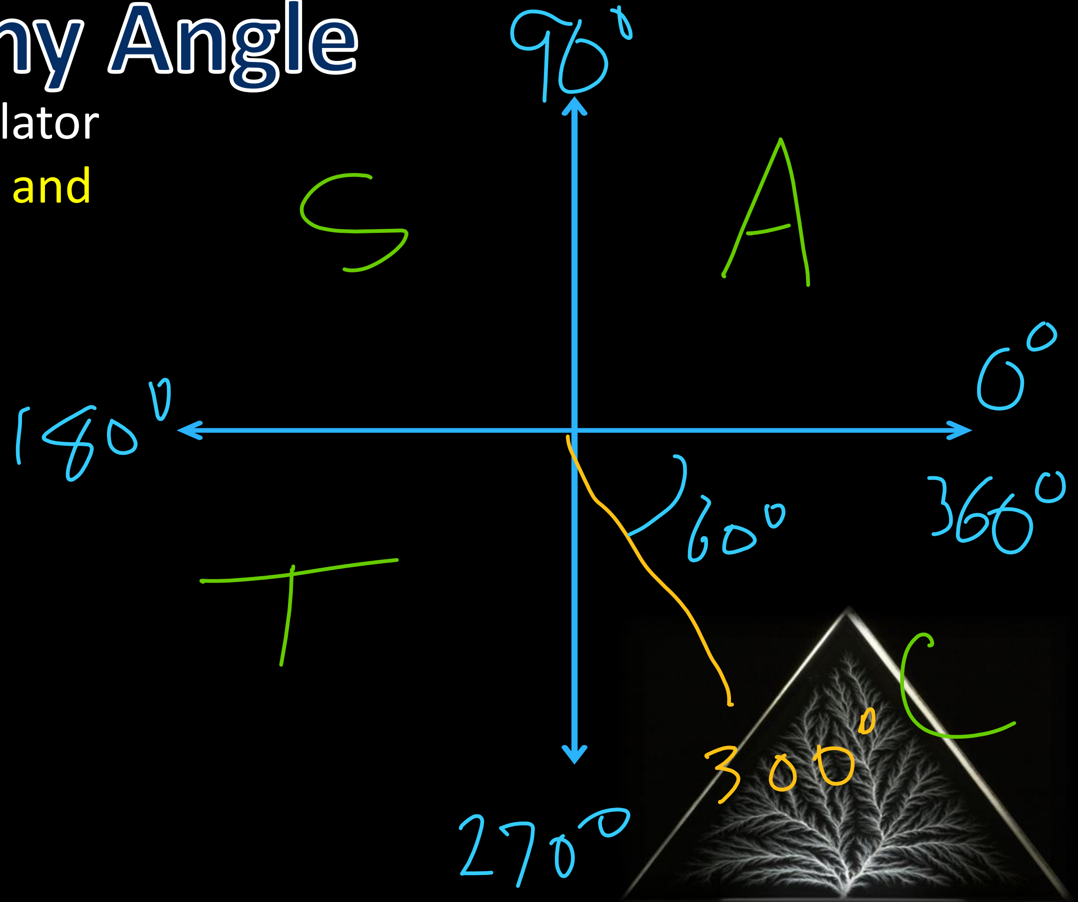
13.3 Evaluate Trigonometric Functions of Any Angle

- Sketch the angle. Then find its reference angle.
- $150°$
  - $180° - 150° = 30°$
- $-\frac{7\pi}{9}$
  - Coterminal ∠ so between 0 and 2π
  - $-\frac{7\pi}{9} + 2\pi$
  - $-\frac{7\pi}{9} + \frac{18\pi}{9} = \frac{11\pi}{9}$
  - Find reference ∠
  - $\frac{11\pi}{9} - \pi$
  - $\frac{11\pi}{9} - \frac{9\pi}{9} = \frac{2\pi}{9}$



13.3 Evaluate Trigonometric Functions of Any Angle

- Evaluate cos(-60°) without a calculator
- Find the coterminal ∠ between 0° and 360°
  - $-60° + 360° = 300°$
- Find reference ∠
  - $360° - 300° = 60°$
- Find cos 60°
  - $\cos 60° = \frac{1}{2}$
- Cos positive in quadrant IV
  - $\cos(-60°) = \frac{1}{2}$



13.3 Evaluate Trigonometric Functions of Any Angle

- Estimate the horizontal distance traveled by a Red Kangaroo who jumps at an angle of 8° and with an initial speed of 53 feet per second (35 mph).
- Range Equation
- $d = \frac{v^2}{32}\sin 2\theta$
- $d = \frac{53^2}{32}\sin\left(2(8°)\right) = 24.2\ ft$

Quiz

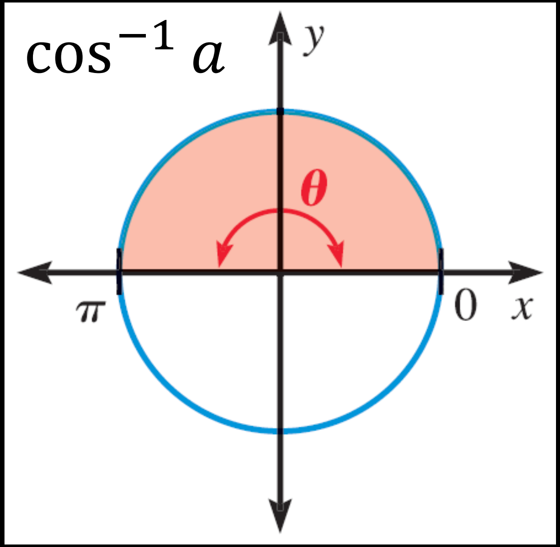
- 13.3 Homework Quiz

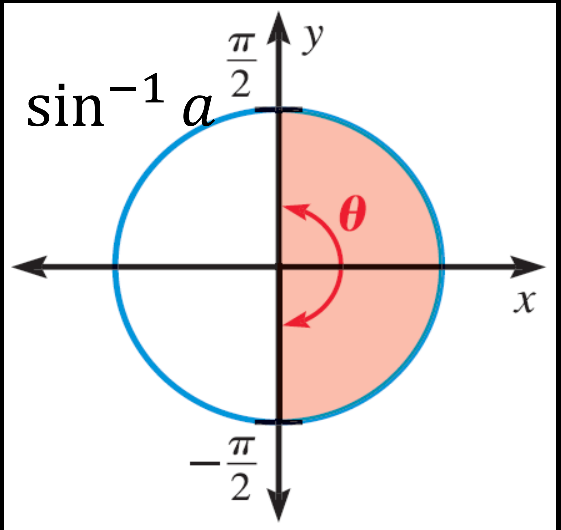
13.4 Evaluate Inverse Trigonometric Functions

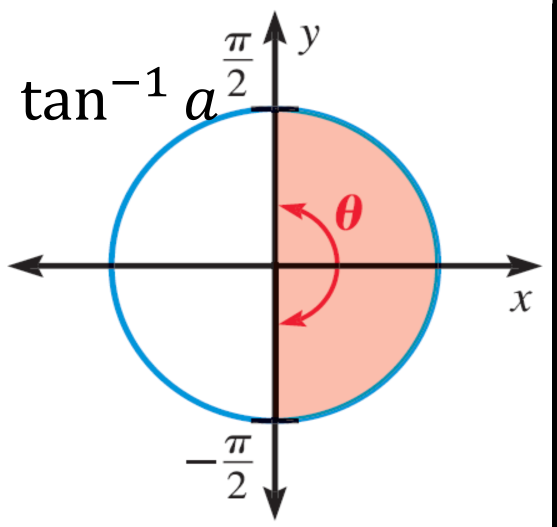
- Find an angle whose tangent = 1
- There are many
  - $\frac{\pi}{4}, \frac{5\pi}{4}, -\frac{3\pi}{4}$, etc.
- In order to find angles given sides (or x and y) we have to define the functions carefully

13.4 Evaluate Inverse Trigo Functions

- Inverse Trig Functions
- $\sin^{-1} a = \theta$
  - $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
- $\cos^{-1} a = \theta$
  - $0 \le \theta \le \pi$
- $\tan^{-1} a = \theta$
  - $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

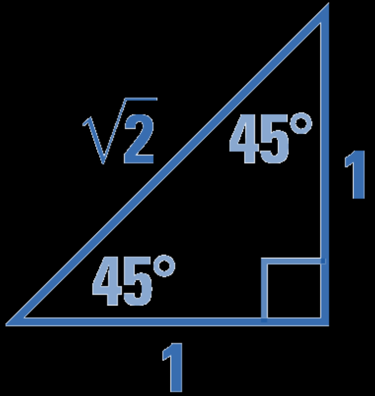




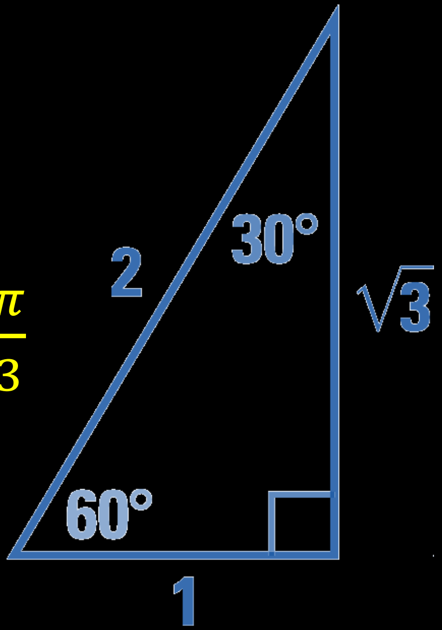


13.4 Evaluate Inverse Trigonometric Functions

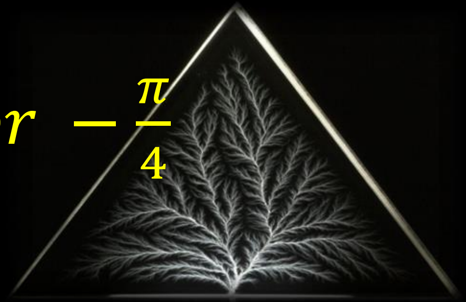
- Evaluate the expression in both radians and degrees.
- $\sin^{-1}\frac{\sqrt{2}}{2}$
  - $\sin^{-1}\frac{1}{\sqrt{2}}$
  - $= 45° \; or \; \frac{\pi}{4}$



- $\cos^{-1}\frac{1}{2}$
  - $= 60° \; or \; \frac{\pi}{3}$

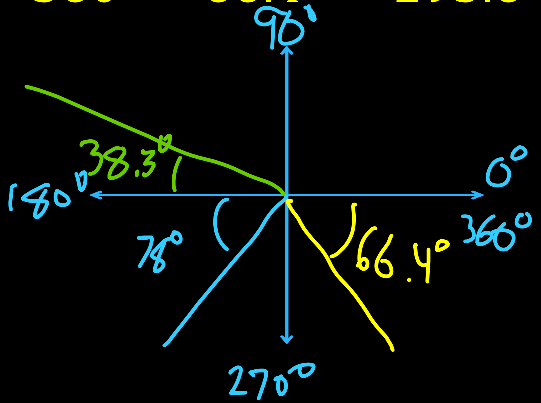


- $\tan^{-1}(-1)$
  - $= -45° \; or \; -\frac{\pi}{4}$

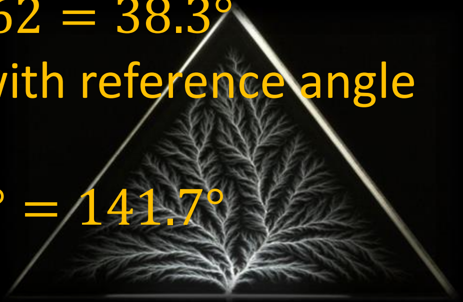


13.4 Evaluate Inverse Trigonometric Functions

- Solve the equation for θ
- $\cos\theta = 0.4;\ 270° < \theta < 360°$
  - $\theta = \cos^{-1} 0.4 = 66.4°$
  - Quadrant IV with reference angle 66.4°
  - $\theta = 360° - 66.4° = 293.6°$

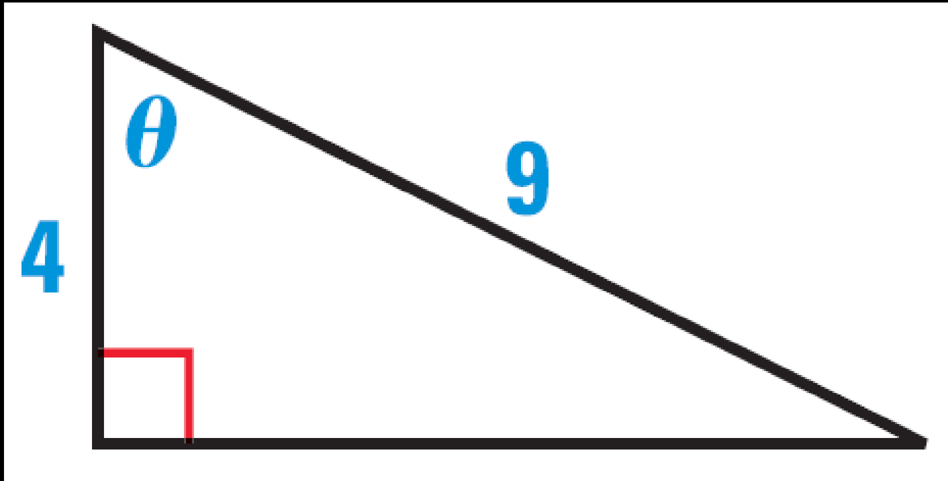


- $\tan\theta = 4.7;\ 180° < \theta < 270°$
  - $\theta = \tan^{-1} 4.7 = 78.0°$
  - Quadrant III with reference angle 78.0°
  - $\theta = 180° + 78.0° = 258°$
- $\sin\theta = 0.62;\ 90° < \theta < 180°$
  - $\theta = \sin^{-1} 0.62 = 38.3°$
  - Quadrant II with reference angle 38.3°
  - $180° - 38.3° = 141.7°$



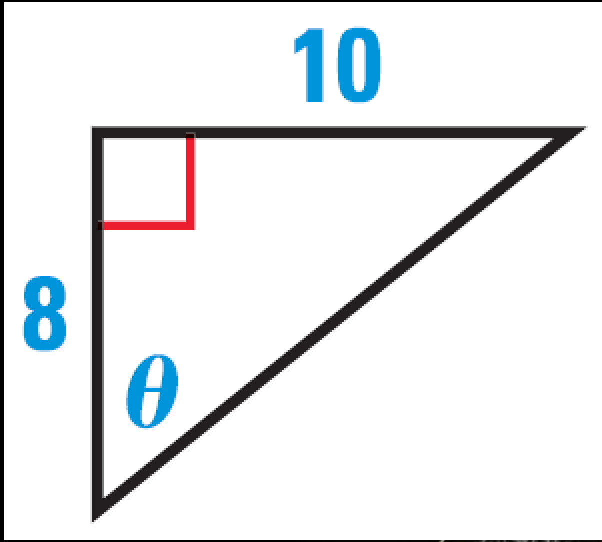
13.4 Evaluate Inverse Trigonometric Functions

- Find the measure of angle θ.
- $\cos\theta = \frac{4}{9}$
- $\theta = \cos^{-1}\frac{4}{9} = 63.6°$



13.4 Evaluate Inverse Trigonometric Functions

- Find the measure of angle θ.
- $\tan\theta = \frac{10}{8}$
- $\theta = \tan^{-1}\frac{10}{8} = 51.3°$



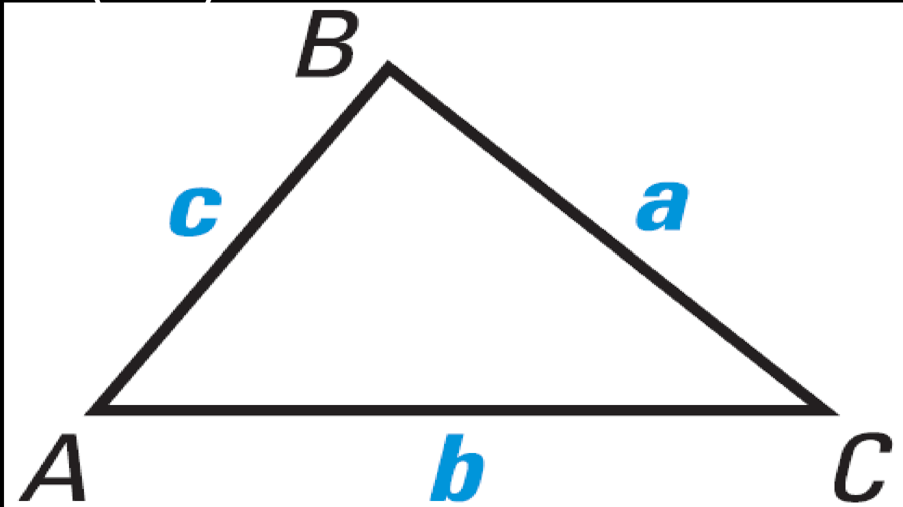
Quiz

- [13.4 Homework Quiz]

13.5 Apply the Law of Sines

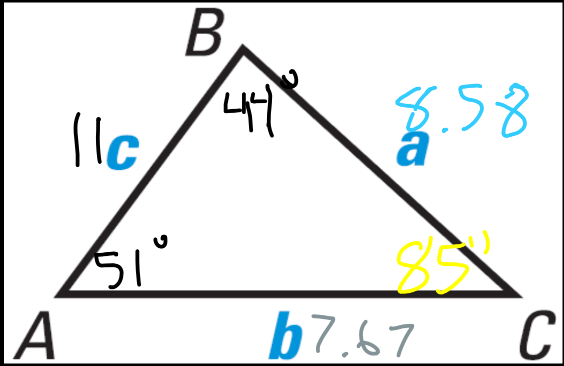
- In lesson 13.1 we solved right triangles
- In this lesson we will solve any triangle if we know
  - 2 Angles and 1 Side (AAS or ASA)
  - 2 Sides and 1 Angle opposite a side (SSA)

- Law of Sines
  - $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



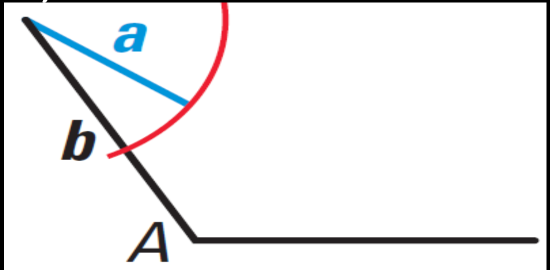
13.5 Apply the Law of Sines

- Solve ΔABC if…
  - A = 51°, B = 44°, c = 11
- $C = 180° - 51° - 44° = 85°$
- $\frac{\sin C}{c} = \frac{\sin A}{a}$
- $\frac{\sin 85°}{11} = \frac{\sin 51°}{a}$
- $a \cdot \sin 85° = 11 \cdot \sin 51°$
- $a = \frac{11 \cdot \sin 51°}{\sin 85°} \approx 8.58$
- $\frac{\sin C}{c} = \frac{\sin B}{b}$
- $\frac{\sin 85°}{11} = \frac{\sin 44°}{b}$
- $b \cdot \sin 85° = 11 \cdot \sin 44°$
- $b = \frac{11 \cdot \sin 44°}{\sin 85°} \approx 7.67$

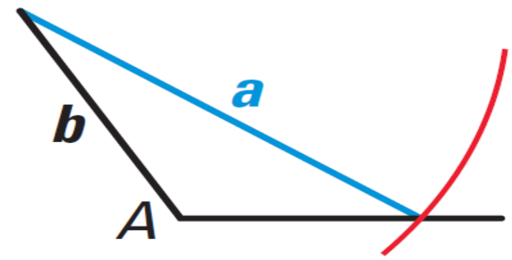


13.5 Apply the Law of Sines

- Indeterminant Case (SSA)
  - Maybe no triangle, one triangle, or two triangles
- In these examples, you know a, b, A
- If A > 90° and…
  - a ≤ b → no triangle

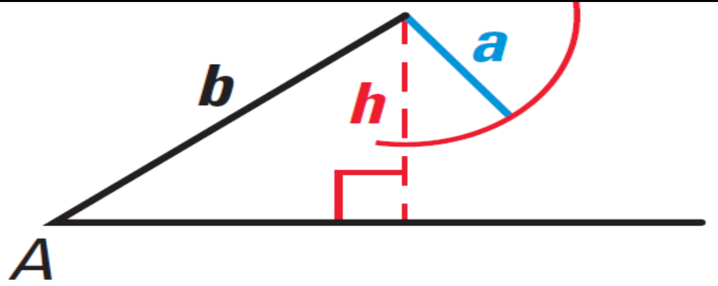


  - a > b → 1 triangle

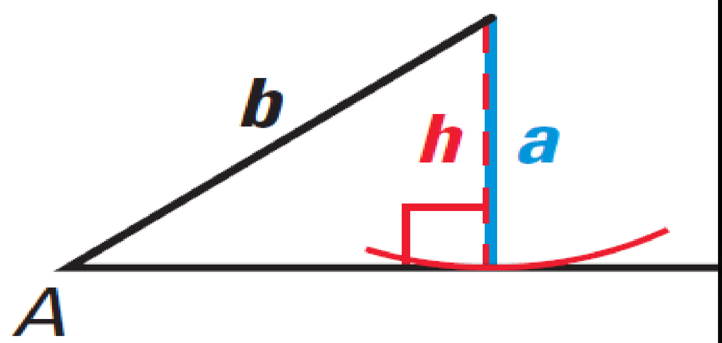


13.5 Apply the Law of Sines

- A < 90° and... (h = b sin A)
  - h > a → no triangle

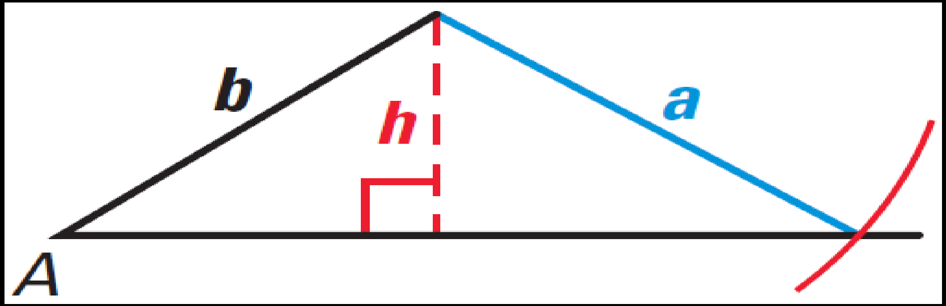


  - h = a → one triangle

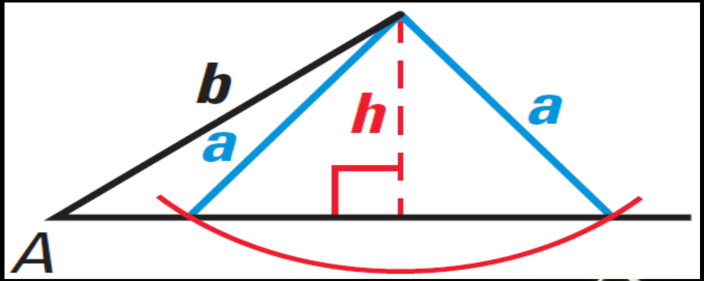


13.5 Apply the Law of Sines

- a ≥ b → one triangle

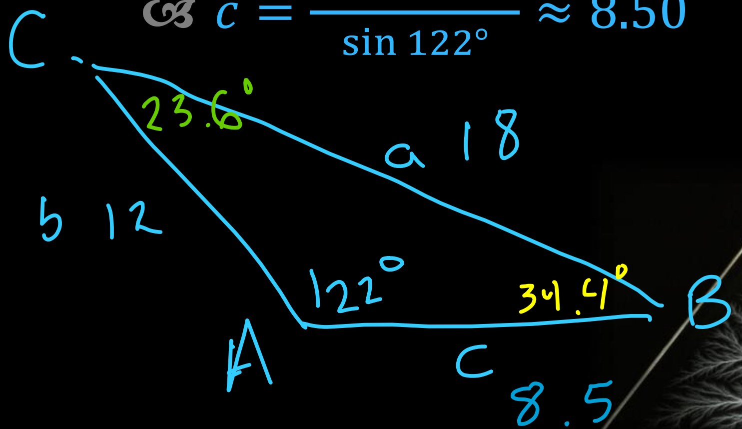


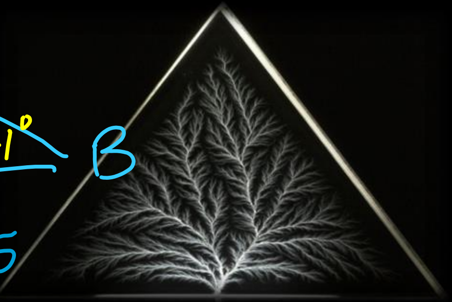
- h < a < b → two triangles



13.5 Apply the Law of Sines

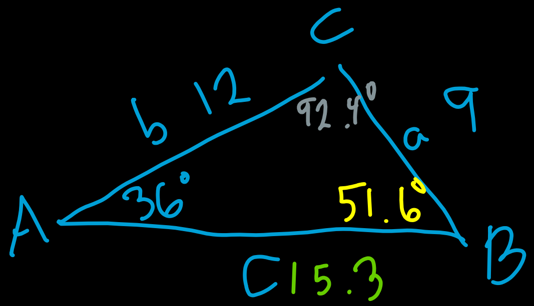
- Solve ΔABC
  - A = 122°, a = 18, b = 12
- $A > 90°$ and $a > b$ → one triangle
- $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$
- $\dfrac{\sin 122°}{18} = \dfrac{\sin B}{12}$
- $18 \sin B = 12 \sin 122°$
- $\sin B = \dfrac{12 \sin 122°}{18} \approx 0.5654$
- $B = \sin^{-1} 0.5654 \approx 34.4°$
- $C = 180° - 34.4° - 122° = 23.6°$
- $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$
- $\dfrac{\sin 122°}{18} = \dfrac{\sin 23.6°}{c}$
- $c \cdot \sin 122° = 18 \sin 23.6°$
- $c = \dfrac{18 \sin 23.6°}{\sin 122°} \approx 8.50$





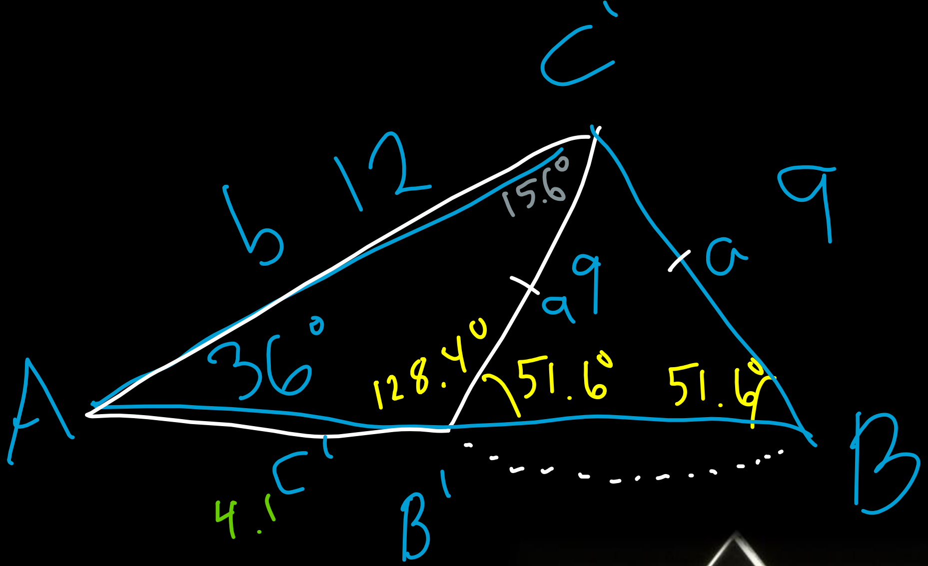
13.5 Apply the Law of Sines

- Solve ΔABC
  - A = 36°, a = 9, b = 12
- $h = b \sin A = 12 \sin 36° = 7.05;$
  $h < a < b \rightarrow$ two solutions
- First solution
- $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$
- $\dfrac{\sin 36°}{9} = \dfrac{\sin B}{12}$
- $9 \sin B = 12 \sin 36°$
- $\sin B = \dfrac{12 \sin 36°}{9} \approx 0.7837$
- $B = \sin^{-1} 0.7837 \approx 51.6°$
- $C = 180° - 51.6° - 36° = 92.4°$
- $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$
- $\dfrac{\sin 36°}{9} = \dfrac{\sin 92.4°}{c}$
- $c \sin 36° = 9 \sin 92.4°$
- $c = \dfrac{9 \sin 92.4°}{\sin 36°} \approx 15.3$



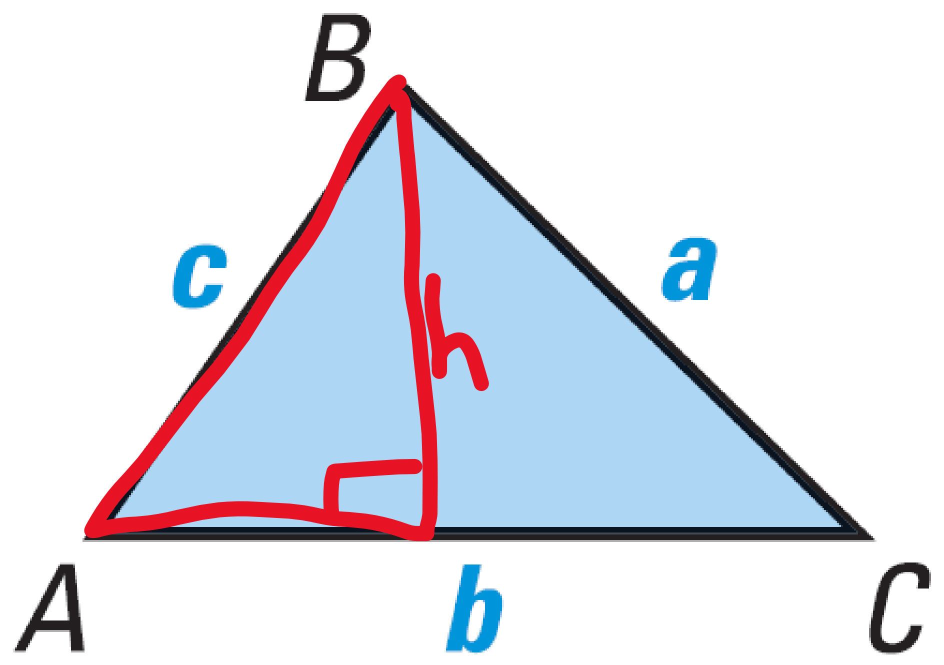
13.5 Apply the Law of Sines

- Solve ΔABC
  - A = 36°, a = 9, b = 12
- Second solution
- To angles for B and they are supplementary
- $B' = 180° - B$
- $B' = 180° - 51.6° = 128.4°$
- $C' = 180° - 128.4° - 36° = 15.6°$
- $\frac{\sin A}{a} = \frac{\sin C'}{c'}$
- $\frac{\sin 36°}{9} = \frac{\sin 15.6°}{c'}$
- $c' \sin 36° = 9 \sin 15.6°$
- $c' = \frac{9 \sin 15.6°}{\sin 36°} \approx 4.1$



13.5 Apply the Law of Sines

- Area of Triangle
  - $Area = \frac{1}{2}bh$
  - $h = c \sin A$
  - $Area = \frac{1}{2}bc \sin A$
- Find the area of ΔABC with…
  - a = 10, b = 14, C = 46°
- $Area = \frac{1}{2}ab \sin C$
- $Area = \frac{1}{2}(10)(14) \sin 46° \approx 50.4$

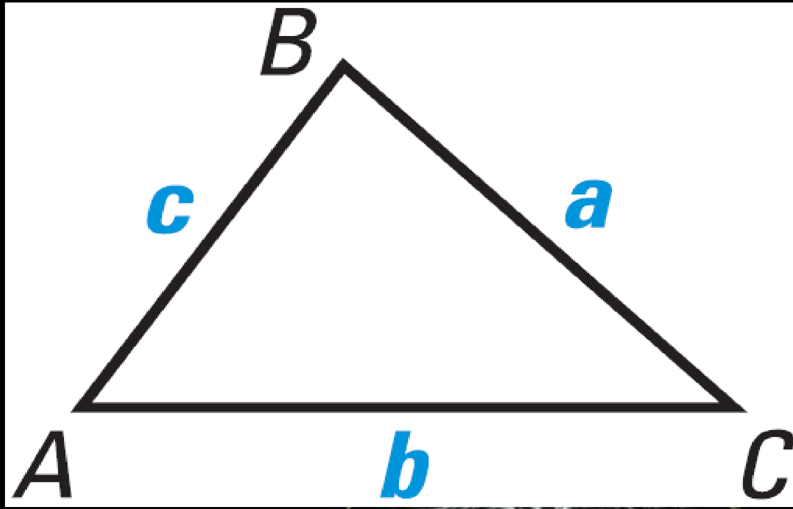


Quiz

- 13.5 Homework Quiz

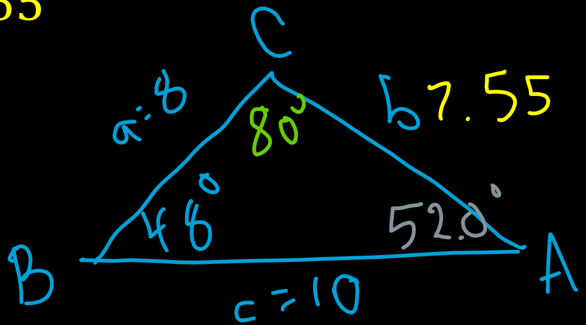
13.6 Apply the Law of Cosines

- When you need to solve a triangle and can't use Law of Sines, use Law of Cosines
  - 2 Sides and Included angle (SAS)
  - 3 Sides (SSS)
- Law of Cosines
  - $a^2 = b^2 + c^2 - 2bc \cos A$
  - $b^2 = a^2 + c^2 - 2ac \cos B$
  - $c^2 = a^2 + b^2 - 2ab \cos C$



13.6 Apply the Law of Cosines

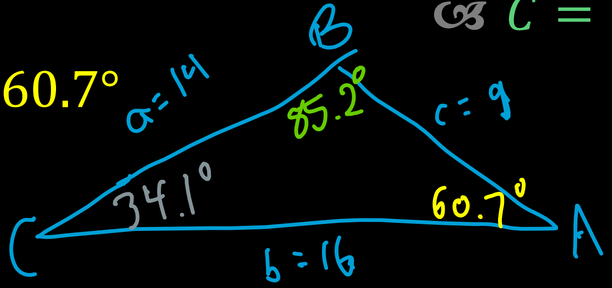
- Solve ΔABC if…
  - a = 8, c = 10, B = 48°
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $b^2 = 8^2 + 10^2 - 2(8)(10)\cos 48°$
- $b^2 = 56.94$
- $b = 7.55$



- $a^2 = b^2 + c^2 - 2ab\cos A$
- $8^2 = 7.55^2 + 10^2 - 2(7.55)(10)\cos A$
- $64 = 57.00 + 100 - 151\cos A$
- $-93 = -151\cos A$
- $0.6159 = \cos A$
- $A = \cos^{-1} 0.6159 \approx 52.0°$
- $C = 180° - 48° - 52.0° = 80°$

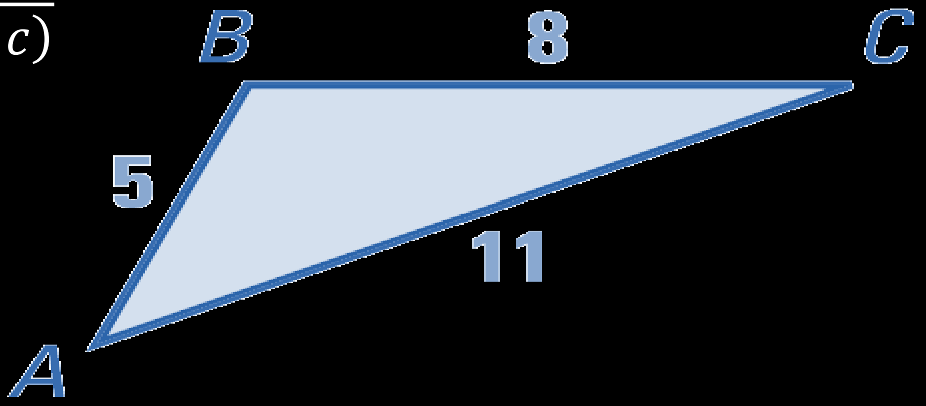
13.6 Apply the Law of Cosines

- Solve ΔABC if…
  - a = 14, b = 16, c = 9
- $a^2 = b^2 + c^2 - 2bc\cos A$
- $14^2 = 16^2 + 9^2 - 2(16)(9)\cos A$
- $196 = 256 + 81 - 288\cos A$
- $-141 = -288\cos A$
- $0.4896 = \cos A$
- $A = \cos^{-1} 0.4896 \approx 60.7°$
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $16^2 = 14^2 + 9^2 - 2(14)(9)\cos B$
- $256 = 196 + 81 - 252\cos B$
- $-21 = -252\cos B$
- $0.0833 = \cos B$
- $B = \cos^{1} 0.0833 \approx 85.2°$
- $C = 180° - 60.7° - 85.2° = 34.1°$



13.6 Apply the Law of Cosines

- Heron's Area Formula
  - $Area = \sqrt{s(s-a)(s-b)(s-c)}$
    - Where $s = \frac{1}{2}(a+b+c)$
- Find the area of ΔABC



- $s = \frac{1}{2}(a+b+c)$
- $s = \frac{1}{2}(8+11+5) = 12$
- $Area = \sqrt{s(s-a)(s-b)(s-c)}$
- $Area = \sqrt{12(12-8)(12-11)(12-5)} = \sqrt{336} = 18.3$

Quiz

- 13.6 Homework Quiz